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THE LATERAL VARIATION OF SOLIDS CONCENTRATION IN HORIZONTAL SLURRY PIPELINE FLOW

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Abstract—Measurements of the solids concentration in the horizontal plane through the pipe axis have been made for slurries of sand in water and solid polystyrene in water. The experiments were conducted in a pipeline of 51 mm dia. At low concentrations, and especially with fine particles, no variation of concentration could be detected in the central 80% of the pipe. As the concentration or the particle diameter increased, a region of reduced concentration near the wall was detected. Experiments at mean velocities of 2 and 3.4 m/s indicated that the migration tendency is insensitive to velocity, for fully suspended flow. The migration tendency seems to result from the dispersive stress effect discovered by Bagnold. Modifications to include this effect in the Schmidt–Rouse model appear to be justified.

BACKGROUND

It has long been recognized that the velocity and concentration distributions in slurry flows are coupled. Any comprehensive theoretical interpretation must deal with these distributions in some way. For example, a simple two-layer model represents the flow as superimposed regions with different concentrations and velocities. The lower-layer concentration is an assumed high value, close to that of a packed bed. Using empirical expressions for the concentration of the upper layer and for the momentum transfer process between the layers, useful predictions of pressure drops can be made.

To proceed beyond this simple model the distributions must be predicted as functions of position. We know that high concentrations of dense solid particles may occur near the bottom of a horizontal pipe. These produce considerable distortion of the velocity distribution. From the two-layer model we also know that the relationship between shear stress and shear rate in slurries differs significantly from that of a single-phase fluid. Thus it is necessary to infer the form of the constitutive relationships from measurements of the distributions. The experimental evidence available for this has been limited.

Previous theoretical studies for horizontal pipe and channel flow have concentrated upon the variation of solids concentration in the vertical direction. In that case gravity (including buoyancy), fluid–particle drag, turbulent diffusion and particle–particle interaction effects must occur. The present study was designed to examine a simpler problem: the variation in a horizontal plane through the pipe axis. Particle diameter, concentration, density and slurry velocity effects were to be investigated experimentally as a contribution to understanding the mechanisms in the flows.

The original model used to explain the concentration variation in the vertical direction is that of Schmidt (1925) and Rouse (1937):

$$\epsilon_s\left(\frac{\partial C}{\partial y}\right) = -Cv_\infty; \qquad [1]$$

C is the volume fraction solids, y is the distance from the bottom of the pipe, v_∞ is the terminal falling velocity of a single particle and ϵ_s is a diffusion coefficient. According to Prandtl (1952), the latter is related to the liquid momentum eddy diffusivity ϵ_L, defined in terms of the turbulent shear stress (τ_{ij}) and the time-average strain rate ($\dot{\gamma}_{ij}$):

$$\dot{\gamma}_{ij} = -(\nabla \mathbf{V} + \nabla \mathbf{V}^T); \quad \rho_L \epsilon_L = \frac{\tau_{ij}}{\dot{\gamma}_{ij}}. \qquad [2]$$

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ϵ_L is a function of position, being given approximately by the expression of Roco & Frasineanu (1977):

$$\frac{\epsilon_L}{Du_*} = 0.073(0.54 + r_1^2)(1 - r_1^2) = 0.073\,g\,(r_1), \qquad [3]$$

where $r_1 = 2r/D$ is the dimensionless radial position and the friction velocity(u_*) is related to the frictional headloss i (m fluid/m pipe) by

$$u_* = \frac{(i\,g\,D)^{0.5}}{2}. \qquad [4]$$

In dilute open-channel flows, concentration profile measurements showed ϵ_s to be greater than ϵ_L by an amount which varied with particle diameter (Raudkivi 1967). Pipeline flow measurements at low concentrations (Sharp & O'Neill 1971; Karabelas 1977) also showed differences of this type.

Slurry pipelines usually employ concentrations which are much higher than those for which [1] was originally proposed and volumetric concentrations in excess of 30% are common. Using a closed rectangular channel which approximated a 1-D flow, Shook *et al.* (1968), measured the C–y distribution by γ-ray absorption. At higher concentrations, values of $\partial C/\partial y$ from [1] were found to be much too low. These could be corrected approximately with a hindered settling multiplier $(1 - C)^n$ on the r.h.s. of [1]. The exponent n was the value from the correlation of Richardson & Zaki given in Wallis (1969). Although this heuristic correction did allow the diffusion model to be extended, quantitative predictions were still difficult because of doubt concerning the appropriate value of ϵ_L to be used in a slurry flow where the mixture density varies with position.

A systematic deviation from this modified diffusion equation was observed with coarse particles. This led to positive values of $\partial C/\partial y$ near the bottom of the pipe in some circumstances. This deviation was attributed to the effect of the dispersive stress discovered by Bagnold (1954).

To incorporate the dispersive stress effect, the diffusion model must first be written as a form of force balance. Including the hindered settling correction the appropriate form [1] would be

$$\left[\frac{3C_D\rho_L\epsilon_s^2}{4dC^2(1 - C)^{2n}}\right]\frac{\partial C}{\partial y}\left|\frac{\partial C}{\partial y}\right| = -(\rho_s - \rho_f)g, \qquad [5]$$

where C_D is the drag coefficient for a single particle settling in the fluid.

The l.h.s. of the equation thus represents turbulent diffusion and mixing effects. This balances the immersed weight of the particles in situations where interparticle contacts are not important.

The dispersive stress evidently contributes an additional term to the force balance. This stress is strongly dependent on concentration: the current state of knowledge has been summarized by Hanes & Inman (1985). This stress requires a finite interparticle shear strain rate and would not exist in a sliding bed of solids. In the latter case, the immersed weight of the particles would be transmitted to the pipe wall by interparticle Coulombic friction. The stress resulting from this type of contact was denoted the "supported load" (Roco & Shook 1983).

Because of these complications, the vertical concentration distributions are difficult to interpret quantitatively. Evidence of a lateral (x-direction) variation has been reported occasionally (Scarlett & Grimley 1974) in conjunction with methods for determining the local concentration. In this case gravity and supported load complications would not arise and the diffusion law would be

$$\left[\frac{3C_D\rho_L\epsilon_s^2}{4dC^2(1 - C)^{2n}}\right]\left(\frac{\partial C}{\partial x}\right)\left|\frac{\partial C}{\partial x}\right| = \frac{-1}{C}\left(\frac{\partial \tau_{sxx}}{\partial x}\right), \qquad [6]$$

where τ_{sxx} is the dispersive stress.

Previous experimental studies of the dispersive stress have employed simple shear devices rather than turbulent pipe flows. Bagnold identified two regions of behavior: the macroviscous and the inertial regimes, distinguished by dimensionless group B:

$$B = \frac{\rho_s d^2 \lambda^{1/2}\dot{\gamma}}{\mu_L} \qquad [7]$$

μ_L is the liquid viscosity and λ is the linear concentration: the ratio of the particle diameter to the mean separation distance between surfaces. In terms of the maximum concentration C_M,

$$\lambda = \left[\left(\frac{C_M}{C}\right)^{1/3} - 1\right]^{-1}. \qquad [8]$$

For $B < 40$, stresses were considered to be transmitted by fluid friction and

$$\tau_{sxx} = \mu_L f(C)\dot{\gamma}. \qquad [9]$$

For $B > 450$, intergranular contact is the transfer mechanism and, according to the results summarized by Hanes & Inman (1985),

$$\tau_{sxx} = \beta \rho_s d^2 F(C)(2 + \alpha)\dot{\gamma} \mid \dot{\gamma} \mid \qquad [10]$$

where

$$\beta = \frac{(1 + e)}{[30(1 - e)]}$$

α is a function of concentration which specifies the contact process, e is the coefficient of restitution of the particles and

$$F(C) = \frac{C^2(2 - C)}{(1 - C)^3}.$$

From an experimental standpoint, the most convenient plane in which to examine the lateral variation is the one through the pipe axis. In this case, x can be replaced by radial distance r.

EXPERIMENTAL PROCEDURE

A laboratory test pipeline, employing recirculating flow, was used for the concentration measurements. The pipeline i.d. was 51 mm. An approach section 180 pipe diameters in length was provided to eliminate the effect of a 90° bend. Previous work had shown bend-induced flow distortions to persist far downstream with particles of low settling tendency. Sharp & O'Neill (1971) reported difficulties of this type which they attempted to correct with straightening vanes.

The volume of the circuit was determined by water displacement so that slurries of a desired concentration could be prepared by adding or removing weighed quantities of solids. To provide a range of particle properties, solid polystyrene particles and sands were used in the experiments. The properties of these are shown in table 1. The concentration C_M of a settled bed of the particles is a measure of the size distribution. Table 1 indicates that all the particles were fairly narrowly sized with the 0.45 mm sand showing the broadest distribution. The breadth of the distribution is important because there will be a tendency for particles to segregate according to size.

During the experiments the temperature was maintained constant by cooling water circulating through a pipe-over-pipe heat exchanger. Bulk velocities were determined with a magnetic flux flowmeter and headlosses were measured with a calibrated variable reluctance pressure transducer. An LSI-11 microcomputer was used for data acquisition.

Experiments were conducted at velocities of 2 and 3.4 m/s to provide a significant variation in u_*. Below 2 m/s the flow of the sand slurries is no longer steady as axial concentrations (waves or dunes) develop. The upper velocity limit was imposed by the available power to the pump and the desirability of using similar velocities for all materials.

Local values of the solids concentration were obtained with a resistivity probe. This device responds to the solids concentration in a region of approx. 1 mm dia. It employs a pair of sensor

Table 1. Properties of particles

Particle	S	d_{50} (mm)	C_D	C_M (%)
Fine polystyrene	1.05	0.3	50	62.4 ± 0.7
Coarse polystyrene	1.06	1.4	2.7	61.7 ± 0.9
Fine sand	2.65	0.19	6.2	66.1 ± 1.3
Medium sand	2.65	0.45	2.7	68.2 ± 0.5
Coarse sand	2.65	0.90	1.1	65.9 ± 1.3

electrodes displaced in the potential gradient created by current flow between two field electrodes. The unique design of the probe eliminates polarization effects which caused earlier, related devices to display a velocity-dependent calibration curve. Details of the measuring system and of the verification of the probe are reported elsewhere (Nasr-El-Din *et al.* 1987). The same probe can be used for particle velocity measurements, using the method described previously by Brown *et al.* (1983) and Gillies *et al.* (1984).

EXPERIMENTAL RESULTS AND DISCUSSION

Figure 1 shows concentrations as a function of distance from the pipe axis for slurries of spherical polystyrene beads of median dia 0.3 mm at a mean velocity U_b of 2.0 m/s. The concentrations reported as C_v are the *in situ* mean values for the whole pipe. These will differ somewhat from the mean values for the lateral planes through the pipe axis.

At the lower mean concentrations, the profiles are flat. However, at the comparatively high concentration of 45% by vol there is evidence of a concentration variation of the type which could be attributed to the dispersive stress. Measurements could not be taken closer to the pipe wall than $2r/D = 0.8$ because of the limitations of the probe's physical size.

Figure 2 shows the corresponding results for slurries of the same particles at a mean velocity of 3.4 m/s. We note again the lack of any dispersive stress effect until high concentrations are reached. We also note the similarity between figures 1 and 2.

Figure 3 shows the lateral concentration variation for slurries of much coarser polystyrene particles at 2.0 m/s. Although not spherical, these particles were approximately isometric with shapes resembling cubes with rounded corners and edges. Figure 3 shows more pronounced migration from the wall than either figures 1 or 2.

Very similar results were obtained at the higher velocity, as shown in figure 4. This is an important result because it confirms the velocity (or headloss) independence which was evident in figures 1 and 2.

These experiments have shown that for nearly neutrally buoyant particles of these sizes, comparatively high mean concentrations are required to produce conditions in which lateral migration can be detected in the central core of the pipe.

The sand slurry experiments were important because of the profound difference in the quantity $(S - 1)$: 1.65 compared to 0.05 or 0.06 for polystyrene. Figure 5 shows the lateral concentration variations observed with the fine sand. Although flatter in the central region, the results resemble those obtained with the 0.3 mm polystyrene. Again, only at the highest concentration is inward particle migration evident.

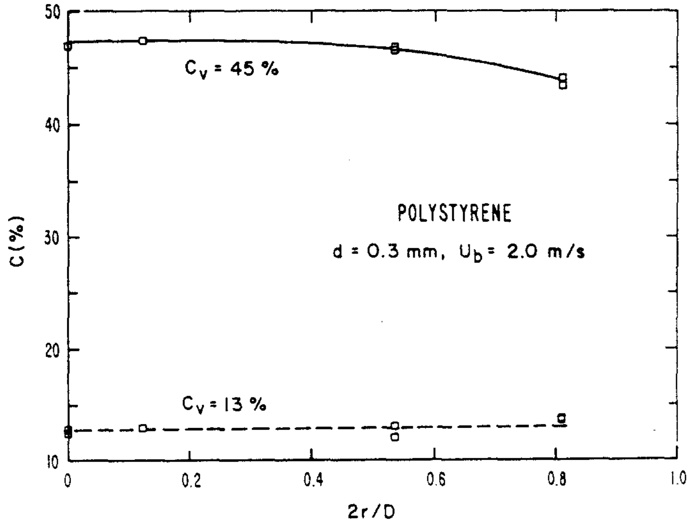

Figure 1. Lateral concentration distribution for fine polystyrene at 2.0 m/s.

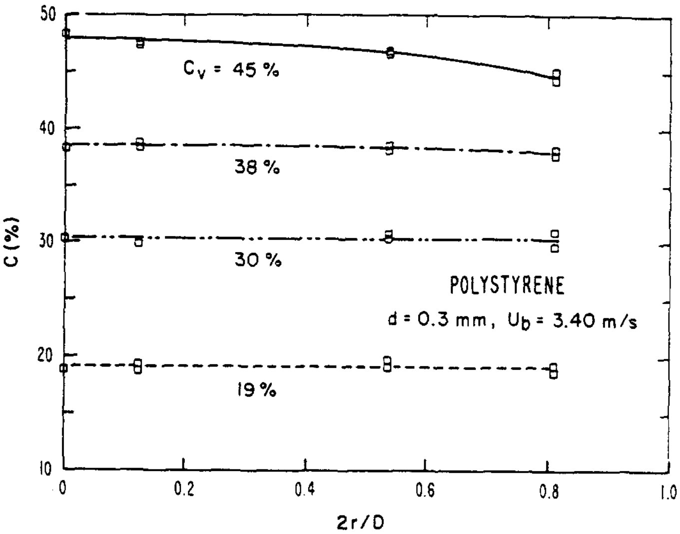

Figure 2. Lateral concentration distribution for fine polystyrene at 3.4 m/s.

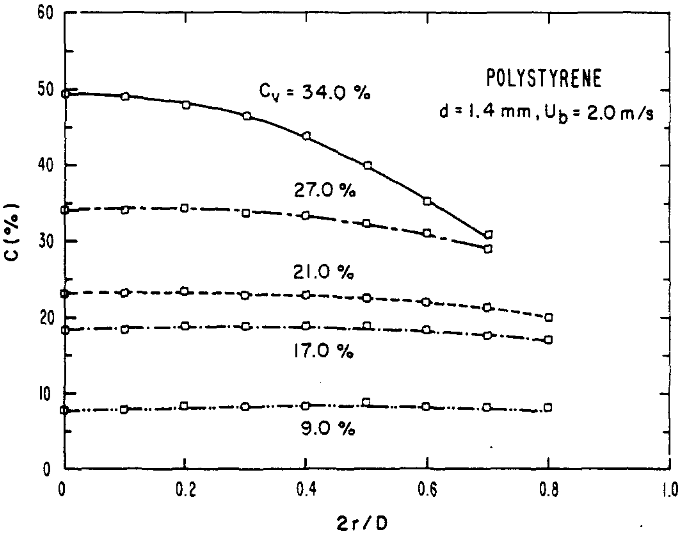

Figure 3. Lateral concentration distribution for coarse polystyrene at 2.0 m/s.

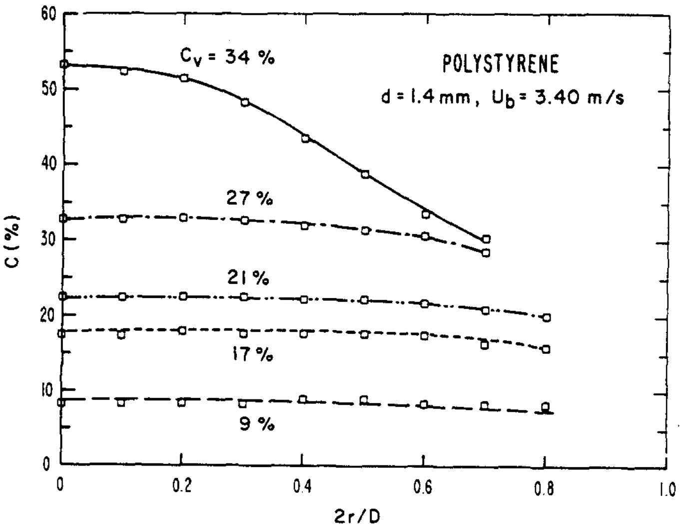

Figure 4. Lateral concentration distribution for coarse polystyrene at 3.4 m/s.

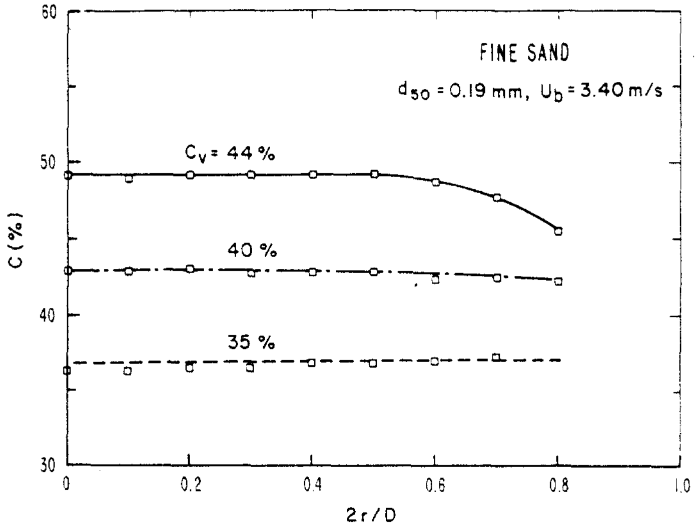

Figure 5. Lateral concentration distribution for fine sand at 3.4 m/s.

Increasing the particle diameter also produces an increase in the lateral migration tendency for the sand particles. This can be seen in figure 6, obtained with 0.45 mm sand. A further increase to a particle diameter of 0.9 mm continues this trend, as shown in figure 7. Here the effect appears to be strong even for concentrations near 25% by vol.

The effect of the dispersive stress can be detected in vertical concentration profiles, where it occurs in combination with gravity. Figure 8 shows the insensitivity of concentration to elevation, resulting from the very low settling velocity of the fine polystyrene particles. Even at the highest concentration the concentration variation is only slightly greater than the scatter in the measurements.

For the coarser particles the results shown in figure 9 are more interesting. There is a significant difference in concentration between the bottom and the top of the pipe at the lowest mean concentration. At this concentration the lateral variation was shown to be minimal in figures 3 and 4. At a mean concentration of 21% by vol, a steeper concentration gradient occurs near the top of the pipe where the migration tendency acts in combination with gravity. Near the bottom of the pipe, gravity is evidently insufficient to maintain a negative concentration gradient and the

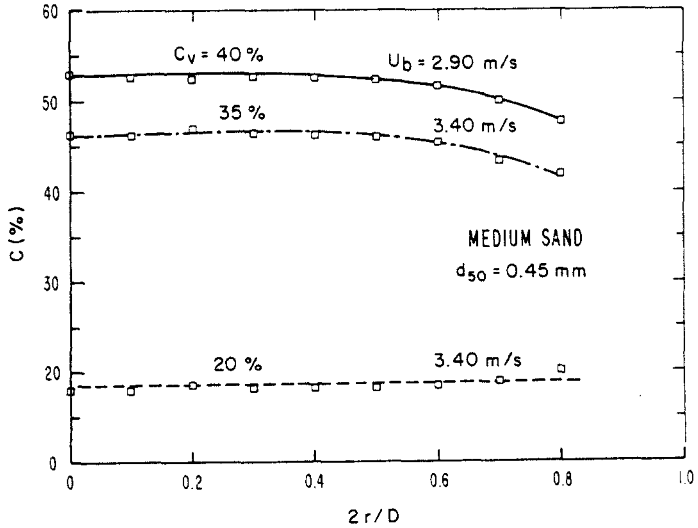

Figure 6. Lateral concentration distribution for medium sand at 3.4 m/s.

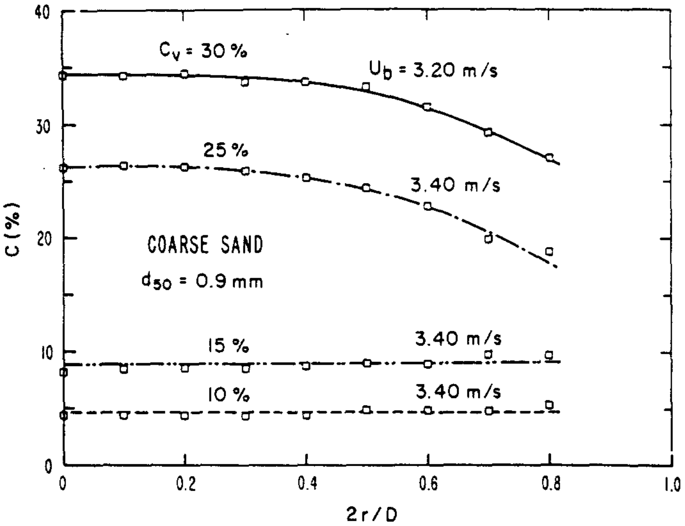

Figure 7. Lateral concentration distribution for coarse sand at 3.2–3.4 m/s.

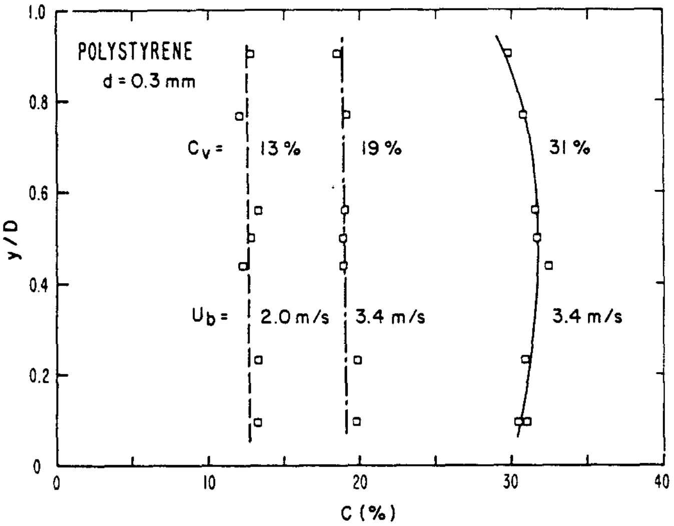

Figure 8. Vertical concentration distribution for fine polystyrene.

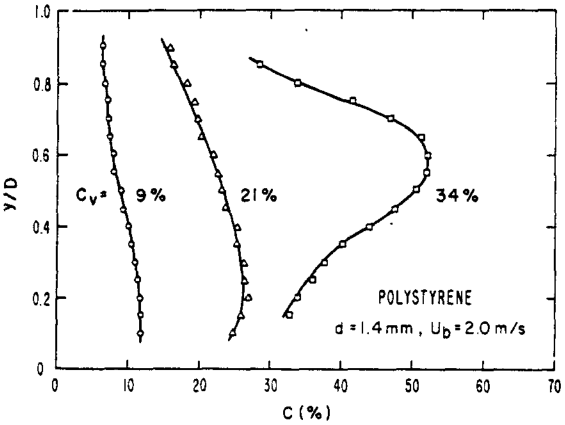

Figure 9. Vertical concentration distribution for coarse polystyrene.

dispersive stress causes a concentration reversal. At the highest concentration the profile is close to symmetrical when the dispersive stress becomes dominant compared to other effects.

Considering the results of figures 1–7, from the velocity independence of the profiles and the assumption that ϵ_s varies as u_*, we conclude that the shearing process must have been inertial in these experiments. We can estimate the time average value of $\dot{\gamma}$ for these flows in the region $r_1 > 0.5$ from the law of the wall in terms of the von Karman coefficient κ:

$$\dot{\gamma} = \frac{2u_*}{\kappa D(1 - r_1)}. \qquad [11]$$

Velocity distribution measurements in the polystyrene slurries showed that κ in the slurry is somewhat higher than the clear fluid value of 0.4. Using estimated $\dot{\gamma}$ values, we find that the Bagnold numbers were < 450 in all cases, and < 40 for the finer particles. It seems Bagnold's criterion does not apply to these turbulent flows.

If the shearing process is inertial, then for $r_1 > 0.5$,

$$\frac{\partial C}{\partial r_1}\left|\frac{\partial C}{\partial r_1}\right| = -\beta_1\left(\frac{d^3 S}{D^3 C_D}\right) C(1 - C)^{2n} \cdot g^{-2}(r_1)\left(\frac{\epsilon_L}{\epsilon_s}\right)^2\left|\frac{\partial}{\partial r_1}\left[\frac{F(C)(2 + \alpha)}{(1 - r_1)^2}\right]\right|, \qquad [12]$$

where

$$\beta_1 = 16.7\frac{(1 + e)}{\kappa^2(1 - e)}$$

and

$$S = \frac{\rho_s}{\rho_L}.$$

Equation [12] indicates that the profiles change rapidly at high values of r_1. This agrees qualitatively with the experiments.

We observe the strong sensitivity to (d/D) predicted by this relationship. Again, this seems to be qualitatively reasonable in the light of the experiments. We also note that the previous indications of the dispersive stress effect have been obtained in small pipelines.

The experimental measurements in figures 1–7 show little lateral variation at low concentrations. This implies that the r.h.s. of [12] should be a strong function of C. However, if one uses previous estimates of ϵ_s (Roco & Shook 1983) and α (Hanes & Inman 1985), [12] does not predict this strong concentration dependence of $\partial C/\partial r_1$. The most likely source of the discrepancy is in the value of n used for the correction term. Although a multiplier of this type is necessary to explain vertical concentration profiles, it seems likely that the n values from the Richardson–Zaki equation are too high.

Equation [12] is rather cumbersome, especially when there are so many unknown quantities. The simpler approximation below may be useful:

$$\frac{-\partial C}{\partial r_1} = \frac{\phi\, C^m}{(1 - r_1)^p}, \qquad [13]$$

with m and $p > 1$ and

$$\phi = \phi\left(\frac{d}{D}, S, C_D \ldots\right).$$

Only the measurements for the coarse polystyrene showed sufficient variation to define m. Using the C values at $r_1 = 0$ and $r_1 = 0.7$, m was found to be approx. 3.

Since the lateral variation is important for all planes passing through the cross section, it was desirable to demonstrate that the experimental conclusions were valid at other locations. Chord-average concentrations in fine sand slurries were measured with a γ-ray absorption apparatus, using a horizontal beam of 1 mm dia. These are compared with the local values, measured at the pipe midplane with the probe, in figure 10. Below about 40% by vol there is little difference between the mean (γ-ray) and the midpoint ($x = 0$) values, implying flat profiles. Above 40%, a significant difference exists and this difference is similar at considerably different y/D values.

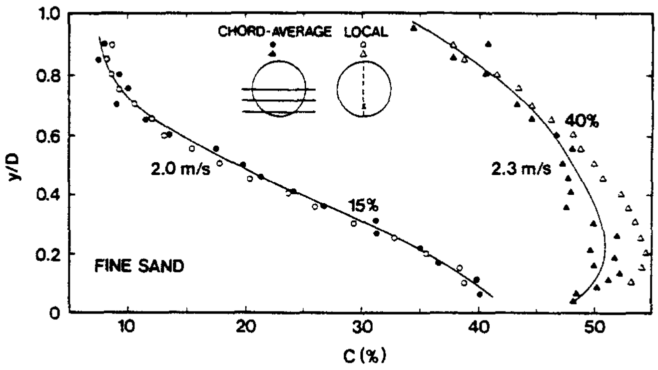

Figure 10. Local (probe) and chord-average concentrations in the vertical plane for fine sand.

It therefore seems likely that [13] could be used at other positions; x would be normalized with the half-width of the pipe in this case.

Figure 10 illustrates the strong effect of gravity on the concentration profiles for sand, in comparison with the nearly neutrally buoyant polystyrene. The combined effects of the dispersive stress and hindered settling begin to become significant as the concentration exceeds 40% by vol. The characteristic reversal occurs near 50%, where the dispersive stress is dominant for these fine particles.

Since it is desirable to have mean values of the correlating coefficients in [13] for the whole pipe, measurements of the type shown in figure 10 are probably more convenient to use in further experimental studies of these effects.

CONCLUSIONS

1. Evidence of concentration variations in the lateral plane through the pipe axis has been obtained for horizontal turbulent slurry flows in a pipeline with 51 mm i.d.
2. Using particles of relative density $S = 1.05$–1.06 and 2.65, the concentration profiles were found to depend upon the particle diameter. The profiles appear to be insensitive to headloss or velocity, at least in the region of fully suspended flow.
3. The particle migration tendency appears to be due to the effect of Bagnold's dispersive stress. However quantitative agreement between pipe flow and simple shear experiments has yet to be demonstrated.
4. A diffusion model related to the usual form appears to have promise as a predictive technique. However, the concentration dependence of the settling tendency term has yet to be established.

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REFERENCES

Bagnold, R. A. 1954 Experiments on the gravity-free dispersion of large solid particles under shear. *Proc. R. Soc.* **A225,** 49–64.

Brown, N. P., Shook, C. A., Peters, J. & Eyre, D. 1983 A probe for point velocities in slurry flows. *Can. J. chem. Engng* **61,** 597–602.

Gillies, R. A., Husband, W. H. W., Small, M. & Shook, C. A. 1984 Some experimental methods for coarse coal slurries. In *Proc. 9th Int. Conf. on Hydraulic Transport of Solids in Pipes*, BHRA Fluid Engineering, Cranfield, U.K., Paper A2, pp. 33–46.

Hanes, D. M. & Inman, D. L. 1985 Observations of rapidly flowing granular materials. *J. Fluid Mech.* **150,** 351–380.

Karabelas, A. J. 1977 Vertical distributions of dilute suspensions in turbulent pipe. *AIChE Jl* **23,** 426–434.

NASR-EL-DIN, H., SHOOK, C. A. & COLWELL, J. 1987 A conductivity probe for measuring local concentrations in slurry systems. *Int. J. Multiphase Flow* **13,** 365–378.

PRANDTL, L. 1952 *Essentials of Fluid Dynamics*. Blackie, London.

RAUDKIVI, A. J. 1967 *Loose Boundary Hydraulics*. Pergamon Press, Oxford.

ROCO, M. C. & FRASINEANU, G. 1977 Computation method of two-phase liquid–solid flow indices in pipeline and channels. *Studii Cerc. Mec. apl.* **36,** 311.

ROCO, M. C. & SHOOK, C. A. 1983 Modeling of slurry flow: the effect of particle size. *Can. J. chem. Engng* **61,** 494–503.

ROUSE, H. 1937 Modern conceptions of the mechanics of fluid turbulence. *Trans. Am. Soc. civ. Engrs* **102,** 463–505.

SCARLETT, B. & GRIMLEY, A. 1974 *Particle Velocity and Concentration Profiles During Hydraulic Transport In a Cirular Pipe*. In *Proc. 3rd Conf. on the Hydraulic Transport of Solids in Pipes*, BHRA Fluid Engineering, Cranfield, U.K., Paper D3, pp. 24–37.

SCHMIDT, W. 1925 Der Massenaustasch in Freien Luft und Verwandte Erscheinungsen. *Probleme kosm. Phys.* **7.**

SHARP, B. B. & O'NEILL, I. C. 1971 Lateral diffusion of large particles in turbulent pipe flow. *J. Fluid Mech.* **45,** 575–584.

SHOOK, C. A., DANIEL, S. M., SCOTT, J. A. & HOLGATE, J. P. 1986 Flow of suspensions in pipelines. *Can J. Chem. Engng* **46,** 238–244.

WALLIS, G. B. *One-dimensional Two-phase Flow*. McGraw-Hill, New York.